# What we can ask to hourly temperature recording. Part II: hourly interpolation of temperatures for climatology and modelling

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**Abstract:** *Several applications in agrometeorology and in biological modelling make use of hourly time series. However, some difficulties arise in finding hourly series of acceptable length, resulting in a limited application of hourly models. In this work, an interpolation model is described which makes use of linear, parabolic and sinusoidal functions between daily maximum and minimum temperatures. Benefits coming from the use of simulated time series are discussed. The errors due to the reconstructed hourly series are small, especially when applications like thermal sums are considered, in which a compensation of single hourly errors takes place. The generation of hourly series allows a good correction of the error committed in mean temperature estimation, when it is calculated by the simple arithmetic average of daily maximum and minimum.*
**Keywords:** *temperature, interpolation*

**Riassunto:** *Le serie temporali orarie hanno diverse applicazioni in agrometeorologia e nella modellistica biologica. Esistono tuttavia difficoltà nel reperire serie orarie di sufficiente lunghezza, tanto che l'uso ne è limitato. Nel lavoro viene descritto un modello che impiega funzioni lineari, paraboliche e sinusoidali per interpolare valori orari a partire dai valori giornalieri di temperatura massima e minima. Vengono quindi discussi i benefici dell'uso delle serie orarie simulate. Gli errori determinati dall'uso di serie orarie ricostruite sono piccoli, specialmente quando si valutano applicazioni come le somme termiche, che consentono una compensazione dei singoli errori orari. La generazione di serie orarie permettere anche di correggere in gran parte l'errore che si commette nella stima delle temperature medie effettuata con la semplice media algebrica di massime e minime giornaliere.*
**Parole chiave:** *temperatura, interpolazione*

In spite of a widespread use of models that make use of daily meteorological values, there are many situations in which hourly resolution is advisable, or even required. This is the case of modelling infection precursors (for example: Rossi *et al.*, 2008; Gleason *et al.*, 1994; Blaeser and Weltzien, 1979); traditionally, chill requirements in phenological models has been expressed in (normalised) chill hours (Ashcroft and Richardson, 1977); Rea and Eccel (2006) found that phenological hourly models based on thermal sums performed better than the equivalent daily ones, even when hourly temperature series were artificially produced from daily maximum and minimum values. Worner (1988) warns about the errors coming from the use of daily values for thermal summations, instead of hourly ones; the author also presents a review of hourly interpolation algorithms developed starting from the sixties.
One problem with the use of hourly temperatures, at least in climate applications, is the limited availability of hourly series, often shorter than the period in which series of minima and maxima are available. In climatic projections, hourly resolution is unavailable from both direct and downscaled model outputs. The present work shows the usefulness of implementing hourly interpolation models, proposing a simple and rapid algorithm. Equations were adapted from the "TM model" (Cesaraccio *et al.*, 2001); in particular, equation (2) was changed and equations (1) and (4) were split to account for possible linear interpolation stretches in the corresponding time bands.
The protocol consists of a first (optional) part of calibration, for the setting of monthly dawn times, hour of minimum, hour of maximum, and sunset times, and of the parameters "c" and "z". Hourly interpolation is carried out with four portions of curves (Fig. 1): parabola/line I, from 0 AM to dawn (same equation of the previous day, from sunset to 11 PM); sinusoid I, from dawn to time of maximum; sinusoid II, from maximum to sunset time; parabola/line II, from sunset to midnight (and continuing till dawn of the following day).
The four equations used are the following:

parab./line I:

$$T(t) = T_{s-1} + \Delta_I \cdot (t + 24 - H_s)^z \qquad 0 \le t \le H_{min} \qquad (1)$$

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sinus I:

$$T(t) = T_{min} + \frac{(T_{max} - T_{min})}{2} \cdot \left[ 1 + sin\left( \pi \frac{t - H_{min}}{H_{max} - H_{min}} - \frac{\pi}{2} \right) \right] \quad (2)$$

$$H_{min} \leq t \leq H_{max}$$

sinus. II:

$$T(t) = T_s + (T_{max} - T_s) \cdot sin\left\{ \frac{\pi}{2} \cdot \left[ 1 + \frac{(t - H_{max})}{(H_s - H_{max})} \right] \right\} \quad (3)$$

$$H_{max} \leq t \leq H_s$$

parab./line II:

$$T(t) = T_s + \Delta_{II} \cdot (t - H_s)^z \quad (4)$$

$$H_s \leq t \leq 23$$

with

$$T_s = T_{\max} - c(T_{\max} - T_{\min+1}) \quad (5)$$

$$\Delta_I = \frac{T_{\min} - T_{s-1}}{(H_{\min} + 24 - H_s)^z} \quad (6a)$$

$$\Delta_{II} = \frac{T_{\min+1} - T_s}{(H_{\min} + 24 - H_s)^z} \quad (6b)$$

where $H_{max}$, $H_{min}$, $H_s$ = times of maximum and minimum temperature and time of sunset, respectively; $T_{max}$, $T_{min}$, $T_s$ = maximum, minimum and sunset temperature, respectively; c: coefficient; z = either 0.5 (parabola - case of "clear late/early night") or 1 (line - case of "cloudy late/early night"). Subscript "-1": day before; subscript "+1": day after. The discrimination between clear and cloudy late night (from midnight to the hour of morning minimum) or early night (from sunset to midnight) was carried out by a calibration on the value of the ratio between daily thermal range ($DTR_d$) and the climatic monthly averages ($DTR_m$):

$$k = \frac{DTR_d}{DTR_m} \quad (7)$$

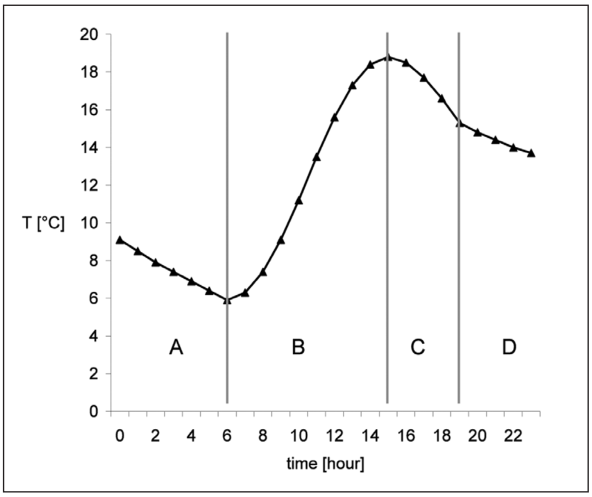


**Fig. 1** – Example of hourly temperature interpolation. A: parabola/line I. B: sinusoid I. C: sinusoid II. D: parabola/line II (see equations in the text).
*Fig. 1 – Esempio di interpolazione oraria delle temperature. A: parabola/retta I. B: sinusoide I. C: sinusoide II. D: parabola/retta II (equazioni nel testo).*

| | $H_{min}$ | | $H_{max}$ | | $H_s$ | | c | |
|---|---|---|---|---|---|---|---|---|
| | S. Michele | Cles | S. Michele | Cles | S. Michele | Cles | S. Michele mean: **0.39** | Cles mean: 0.38 |
| JAN | 8 | **8** | 15 | **15** | 17 | **17** | 0.36 | 0.36 |
| FEB | 8 | **8** | 16 | **15** | 18 | **18** | 0.37 | 0.38 |
| MAR | 7 | **7** | 15 | **15** | 18 | **18** | 0.26 | 0.28 |
| APR | 6 | **6** | 15 | **15** | 19 | **19** | 0.36 | 0.37 |
| MAY | 5 | **5** | 15 | **15** | 20 | **19** | 0.46 | 0.36 |
| JUN | 5 | **5** | 15 | **15** | 20 | **20** | 0.45 | 0.46 |
| JUL | 5 | **5** | 15 | **15** | 20 | **20** | 0.44 | 0.47 |
| AUG | 6 | **6** | 15 | **15** | 19 | **19** | 0.35 | 0.39 |
| SEP | 6 | **6** | 15 | **15** | 18 | **18** | 0.30 | 0.34 |
| OCT | 7 | **7** | 15 | **15** | **18** | 17 | 0.49 | 0.30 |
| NOV | 8 | **7** | **15** | 14 | **17** | 17 | 0.41 | 0.40 |
| DEC | 8 | **8** | 15 | **15** | **17** | 17 | 0.41 | 0.47 |

**Tab. 1** – Calibration of parameters for TM model. $H_{min}$: hour of minimum temperature; $H_{max}$: hour of maximum temperature; $H_s$: hour of sunset; c: coefficient for the determination of $T_s$ (sunset temperature – see equation 5). Values used in the "unified" setting are highlighted.
*Tab. 1 – Calibrazione dei parametri per il modello TM. $H_{min}$: ora della temperatura minima; $H_{max}$: ora della temperatura massima; $H_s$: ora del tramonto; c: coefficiente per la determinazione di $T_s$ (temperatura al tramonto – vedi formula 5). I valori impiegati nell'impostazione "unificata" sono evidenziati.*

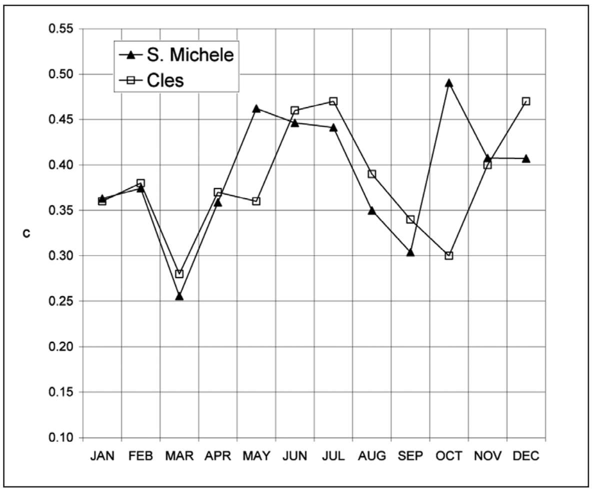


**Fig. 2** – Values of the coefficient "c" for the assessment of sunset temperature – see equation 5 - calibrated monthly over a 27-year period (1983 – 2009), stations of S. Michele and Cles.
*Fig. 2 – Valori del coefficiente "c" per la stima della temperatura al tramonto - vedi formula 5 - calibrato mensilmente su un periodo di 27 anni (1983 – 2009), stazioni di S. Michele e Cles.*

which gave for both stations and for both cases ("late night" and "early night") an optimal value of 1.5: for lower values the linear equation allows for better interpolation than the parabolic one.

A monthly differentiation is important for times of minimum ("dawn") and for times of change of the equation from sinusoid II to parabola/line II ("sunset"), secondarily for the time of maximum, which, for mean hourly recordings, peaks in frequency (with few exceptions) at 3 PM (that is, in the previous 60 minutes). $H_s$ was calibrated as the time of the day more frequently associated to the largest temperature fall in the afternoon. Also the parameter "c", necessary for the definition of the sunset temperature $T_s$, can be easily calibrated on a monthly basis. The authors of the TM model proposed a mean value for "c" of 0.39, which is fully confirmed by the calibration carried out on the two 27-year long series of S. Michele (210 m a.s.l., valley bottom), and Cles (650 m a.s.l., hill side), in Trentino (Tab. 1 and Fig. 2). With the exception of a couple of months, a good agreement can be observed for the calibration of "c" between the two stations. As regards the calibration of timing, in general, a difference of one hour is not dramatic, corresponding to shifts in the discontinuities of the curve type at a time when either the rate of temperature change is at its minimum ($H_{min}$ and $H_{max}$ times), or both curves are decreasing ($H_s$ time), resulting in limited errors.

The introduction of linear interpolation of night values as alternative to the parabolic one allows for an improvement of both mean and absolute errors, thanks to the calibration of the parameter k (eq. 7). Indeed, when night radiation loss is

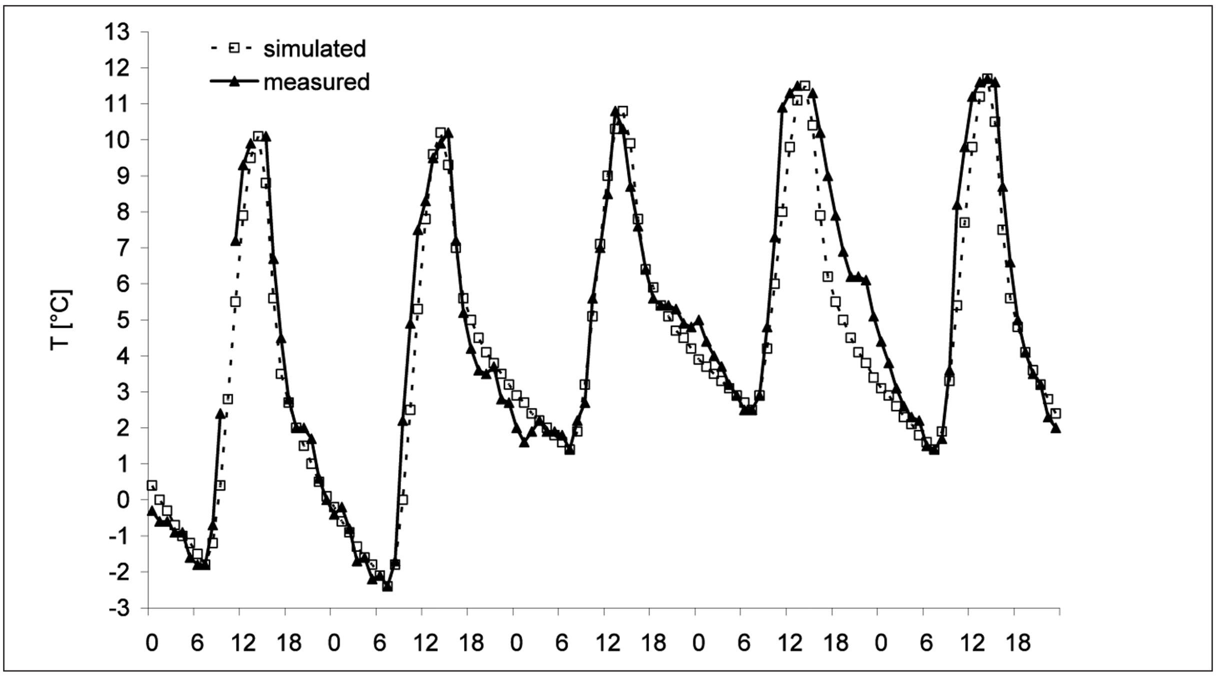


**Fig. 3** – Example of hourly interpolation, compared with measured values (S. Michele, 24th – 28th January 1983).
*Fig. 3 – Esempio di interpolazione oraria, comparata con i dati misurati (S. Michele, 24 – 28 gennaio 1983).*

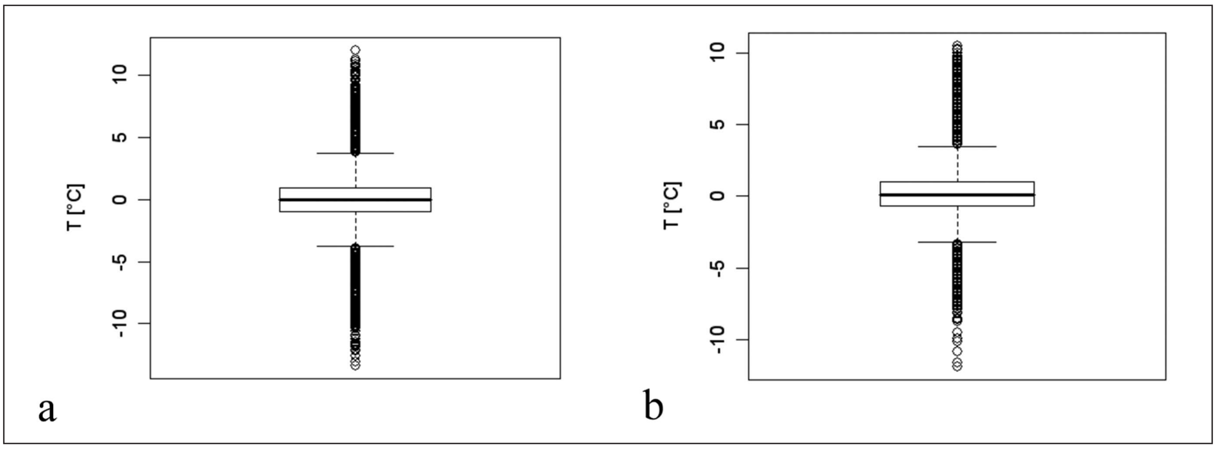


**Fig. 4** – Boxplots of estimation errors of hourly temperature (estimated – measured), 1983-2009. a) S. Michele; b) Cles.
*Fig. 4 – Boxplot dell'errore di stima della temperature oraria (stimata – misurata), 1983-2009. a) S. Michele; b) Cles*

inhibited (as in the case of overcast or just cloudy sky), the temperature decrease is irregular and, in general, there is no clear sign of a concavity in the corresponding line.
For simulating the case of unavailability of hourly series, a "unified" set of parameters was chosen. It can be seen that there are no large differences in the two series between $H_{min}$, $H_{max}$ and $H_s$, so the "smoothest" set of monthly $H_{min}$ and $H_s$ was chosen (see Tab. 1), while $H_{max}$ was fixed to the constant value of 15 (3 PM) and c to the constant value of 0.39.
The application of the interpolation algorithm (excerpt in Fig. 3) yields a mean error of -0.01 / +0.04 °C and a mean absolute error of 1.29 / 1.14 °C, for S. Michele and Cles, respectively. There

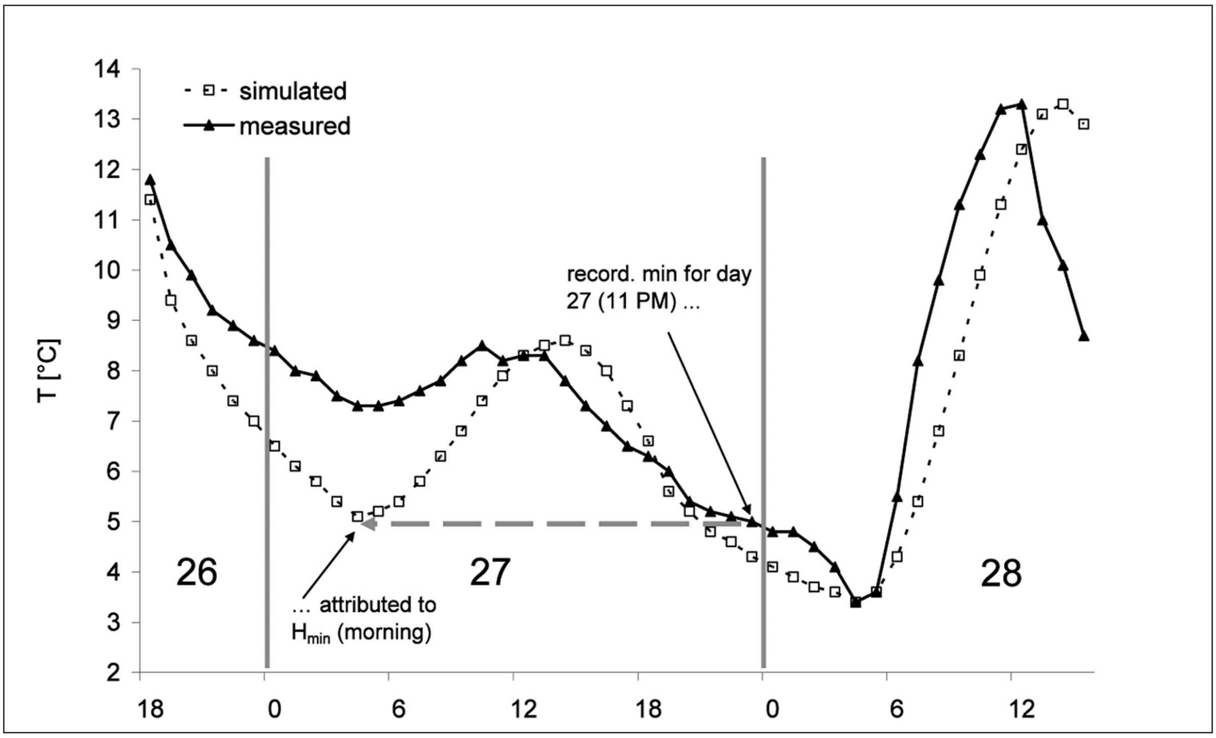


**Fig. 5** – Example of interpolation error due to the erroneous attribution of the time of minimum temperature by the interpolation algorithm, when no restriction is applied to the possible timing of occurrence of minima. Cles, 26th – 28th May 1984.
*Fig. 5 – Esempio di errore di interpolazione dovuto all'erronea attribuzione dell'ora della temperatura minima, quando non è applicata alcuna restrizione sugli orari possibili di occorrenza delle minime. Cles, 26 – 28 maggio 1984.*

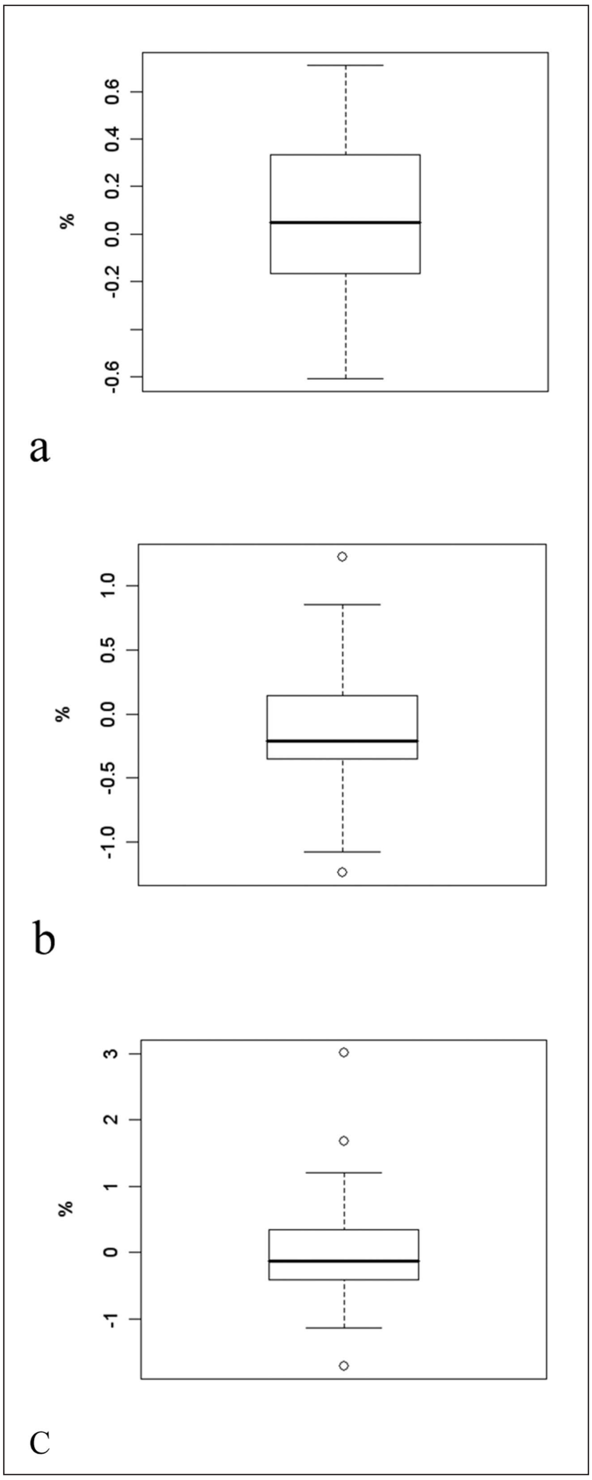


**Fig. 6** – Boxplots of percentages of errors of the hourly thermal sums (1st Jan – 31st Dec., 1983 – 2009), from the comparison with measured hourly values – Station of Cles. a) $T_{soglia}$ = 0°C. b) $T_{soglia}$ = 5°C. c) $T_{soglia}$ = 10°C.
*Fig. 6 – Boxplot degli errori percentuali di stima delle somme termiche orarie (1° genn. – 31 dic., 1983 – 2009) determinate dal confronto con i valori orari misurati – Stazione di Cles. a) $T_{threshold}$ = 0°C. b) $T_{threshold}$ = 5°C. c) $T_{threshold}$ = 10°C.*

are also single major errors, as can be seen from fig. 4, but the Inter-Quantile Range (IQR) remains narrow. During the trials, a few considerable errors in the records have been detected and corrected, but it is highly probable that still a few cases of disagreement between modelled and measured temperatures could be attributed to errors in the database, rather than to the interpolation itself. The introduction of the "linear" option in eqs. 1 and 4 allows the removal of a non-negligible negative bias of estimation, which would have required an offset of the interpolation curves in the range of 0.5 - 0.6 °C.

The interpolation algorithm is intrinsically unbiased in the reproduction of both minimum and maximum daily values. Indeed, it functions pretty well, but it rather strictly reproduces the same pattern of temperature trend for every day, even with the timings of curve discontinuity ($H_{min}$, $H_{max}$ and $H_s$) changing from month to month and the night-time curve type changing according to the case. One assumption is that the time of minimum falls in a time band ranging from 5 to 8 AM, according to the month. The ideal minimum series should be built by calculating minimum temperature in that period of the day, so temperature strongly decreasing after sunset would not contribute to the search for minimum of that day (a well-known problem, see e.g. Parton and Logan, 1981, and Wann *et al.*, 1985). This has its physical grounds; however, in most meteorological databases, this is not the case, minimum temperatures being just the lowest measured in each day from 0 AM to 12 PM. In the first part of this investigation, Eccel (2010) calculated that days when minimum temperatures fall after midday account for, roughly, one-fifth of the total. Now, the results above refer to "blind" daily series, whose corresponding hourly series were not considered in building minimum and maximum series. A check on the major interpolation errors shows that many of them are due to the wrong attribution of the time of occurrence of maximum and, above all, of minimum daily temperature (the mechanism is exemplified in Fig. 5). The absolute error of the interpolation could be reduced further by using maximum and minimum series created with the abovementioned "filter" on the time bands for the search of minima.

It may be argued that the mean absolute error of the interpolation, larger than 1 °C, is still non-negligible. In order to assess the cumulative error that can arise from the use of an hourly series, a

test was carried out to compare values of thermal sums originated by interpolated data with the true ones, calculated with measures. In many agro-meteorological applications (especially vegetal and parasite developmental models), the use of hourly series is associated to the calculation of thermal sums, so this control is particularly delicate and useful. Three thresholds were considered: 0°C, 5°C, and 10°C. Results show that errors increase with the threshold, but they remain fully acceptable, mean values being as low as -0.13 % ± 0.51 % (period: 1983-2009) for a threshold temperature of 10°C (Fig. 6).
Finally, it may be interesting to assess the improvement of the creation of simulated hourly temperature in a series to the correct estimation of mean daily temperature. In absence of other information, mean temperature is usually calculated as the average between minimum and maximum daily values. But, when the average can be calculated from hourly values, the error committed when averaging $T_{min}$ and $T_{max}$ is evident: a mean absolute error of 0.85 ±0.73/ 0.72 ±0.52°C, respectively for S. Michele and Cles (for the usual period 1983-2009), and (remarkable) biases of, respectively, +0.61 ±0.94/ +0.57 ±0.68°C. On the other hand, the production of the hourly interpolation can lower such high biases of one order of magnitude, as reported above, even if it cannot decrease the standard deviation of daily errors (0.79 / 0.58 °C, respectively for S. Michele and Cles).
The reported results can be certainly refined, by taking into consideration more than two stations, possibly displaying more pronounced differences in climatic features. This investigation, and the analyses of possible differences from one site to another, make up the core of future development of this work. Nevertheless, the similarity in the results of the calibration trials of the two analysed series encourages its generalization and the extension to sites where no hourly records are available. So evident are the advantages coming from the use of simulated hourly series, and so easy the application of interpolation algorithms, that we warmly feel to encourage their use.

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*Note: the R codes for hourly interpolation are available on demand.*

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